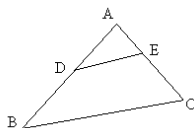


FORTY-THIRD ANNUAL
AUGUSTA UNIVERSITY
MATHEMATICS CONTEST MARCH 3, 2017
TEAM PROBLEMS

1. Consider the odd integers from 1 to 199, i.e., $\{1, 3, 5, \dots, 195, 197, 199\}$. What is the fraction of even digits among all the digits needed to write these 100 odd integers?
2. What is the value of $\sqrt{\frac{8^{10} + 4^{10}}{8^4 + 4^{11}}}$?
3. In triangle ABC , $AD = DB$, DE is parallel to BC , and the area of triangle ABC is 40. What is the area of triangle ADE ?



4. Find the number of real solutions of the equation

$$x^9 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + 1 = 0$$

5. Four pirates found a chest of gold coins. They agreed to divide the treasure equally when they got to port. Before reaching port, the captain secretly took $1/3$ of the coins. After him, the first mate took $1/3$ of what was left. Then the bosun snuck in, taking $1/3$ of what was left. The cabin boy did the same. At port, each pirate took 400 coins. How many coins were in the chest originally?
6. What is the value of the sum

$$\log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \dots + \log_{10}(\tan 88^\circ) + \log_{10}(\tan 89^\circ)?$$

7. Two baseball teams (team A and team B) are playing a best-of-five game series (that is, once one of the teams wins three games, the series is over and the team winning three games is the champion) Team A is the superior team and they will win any given game against team B with probability $2/3$. Thus, find the probability that team A wins the series.
8. Two squares are inscribed in an isosceles right triangle. The first square has two of its vertices on the hypotenuse and the other two vertices on the two legs of the triangle. The second square has one vertex on the vertex of the right triangle and another on the hypotenuse. Find the ratio of the area of the first square to the area of the second square.

